DYNAMICS OF A TURBULENT THERMIC OF JET TYPE IN THE ONE-DIMENSIONAL APPROXIMATION

V. P. Kabashnikov

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The solution of the approximate equations of motion strongly extended thermics is obtained. A general relation is obtained between the integral momentum and buoyancy.

Aircraft engines release a large quantity of heated combustion products into the atmosphere. It is evident that their flow has very characteristic features of a jet in the boundary-layer approximation, on the one hand [1]. On the other, buoyancy forces begin to influence the dynamics and structure of the exhaust gas after a sufficiently long period of time. Therefore, the exhaust-gas wake behind an airplane in vertical take-off may be regarded as a jet-type thermic after a sufficient period of time [2, 3]. The principal feature of the wake is the high longitudinal homogeneity of its parameters in the initial period of its evolution.

A series of general results on the dynamics of thermics was obtained in [4-6] — regarding, in particular, the possibility of existence of self-similar conditions and the asymptotic laws of motion at large times. In [7-10], analytical expressions describing the nonsteady motion of two-dimensional thermics in some limiting cases were obtained. A series of results for nonextended thermics were obtained using equations for the mean parameters over the thermic volume [6, 11-14]. In view of the considerable analytical difficulties, methods of numerical investigation of thermics have also been developed [15-18].

In the present work, a general relation is obtained between the integral (over the volume) vertical momentum and the buoyancy of the thermic. Attention focuses mainly on turbulent thermics, which are almost homogeneous in the vertical direction at the instant of their formation. The approximation of one-dimensional gas dynamics is used here, employing mean quantities across the jet.

Considering subsonic flow without chemical reactions, and assuming isobaric conditions across the jet, the hydrodynamic equations are written in the following form, disregarding the molecular viscosity and heat conduction

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v_k}{\partial x_k} = 0, \quad (1)$$

$$\frac{\partial \rho \Delta \varkappa_m}{\partial t} + \frac{\partial \rho v_k \Delta \varkappa_m}{\partial x_k} + \rho v_k \frac{\partial \varkappa_{m\infty}}{\partial x_k} = 0, \quad (2)$$

$$\frac{\partial \rho v_i}{\partial t} + \frac{\partial \rho v_i v_k}{\partial x_k} - g_i \Delta \rho = 0, \quad (3)$$

$$\frac{\partial \rho c_p \Delta T}{\partial t} + \frac{\partial \rho c_p v_k \Delta T}{\partial x_k} + \rho v_k \frac{\partial c_p T_\infty}{\partial x_k} - \rho v_k g_k = 0. \quad (4)$$

Suppose that the deviations in temprature ΔT, molecular weight $\Delta\mu$, and density $\Delta\rho$ inside the thermic from the corresponding parameters at the same height in the surrounding medium are sufficiently small, and the specific heat c_p is constant. If the dimensions of the thermic are sufficiently small, so that the Weisel–Brent frequency may be regarded as constant within the height range occupied by the thermic [6], the following result is obtained under the given assumptions, integrating Eqs. (2)-(4) over the volume between

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infinite limits and taking into acount that the velocity and excess density at infinity aer zero

$$\frac{\partial}{\partial t}\tilde{\Pi} = -N^2\tilde{J}, \tag{5}$$

$$\frac{\partial}{\partial t}\tilde{J} = \tilde{\Pi}, \tag{6}$$

where $\tilde{J}$ is the integral (over the volume) vertical component of the momentum

$$\tilde{J} = \int \rho_\infty u dV, \tag{7}$$

$\tilde{\Pi}$ is the integral (over the volume) buoyancy

$$\tilde{\Pi} = \int \rho_\infty \Omega dV, \tag{8}$$

$$\Omega = -g\frac{\Delta\rho}{\rho_\infty} = g\left(\frac{\Delta T}{T_\infty} - \frac{\Delta\mu}{\mu_\infty}\right), \tag{9}$$

μ is the molecular weight of the gas mixture; N is the Weisel-Brent frequency [6]

$$N^2 = g\left(\frac{g}{c_p T_\infty} + \frac{1}{T_\infty}\frac{\partial T_\infty}{\partial z} - \frac{1}{\mu_\infty}\frac{\partial \mu_\infty}{\partial z}\right). \tag{10}$$

It follows from Eqs. (5) and (6) that the integral momentum and buoyancy perform non-damping oscillations with frequency N such that the quantity $\tilde{\Pi}^2 + N^2J^2$ is conserved over time, with stable stratification of the atmosphere ($N^2 > 0$). The damping of the mean (over the thermic volume) velocity and excess density is determined by the rate of increase in its volume.

Below, a turbulent axisymmetric vertical jet is considered, assuming satisfaction of all the conditions adopted in deriving Eqs. (5) and (6), except for the assumption of finite vertical dimensions. After averaging Eqs. (1)-(4) with respect to the turbulent pulsations, the resulting equations are averaged over the finite jet cross section, assuming that the dynamic, thermal, and concentrational widths of the jet are the same. As an example, the result of double averaging of Eq. (4) is given

$$\frac{\partial \rho_\infty \langle \overline{\Delta T} \rangle S}{\partial t} + \frac{\partial \rho_\infty \langle \overline{\Delta T \cdot u} \rangle S}{\partial z} + \rho_\infty \langle \bar{u} \rangle S\left(\frac{\partial T_\infty}{\partial z} + \frac{g}{c_p}\right) = 0, \tag{11}$$

where

$$\langle \bar{L}(z, t) \rangle = S^{-1}(z, t)\int_0^{R(z,t)} 2\pi r \langle L(z, r, t) \rangle dr. \tag{12}$$

Assuming approximately that the mean of the product is equal to the product of the means, and omitting the averaging symbols and the subscript ∞ for the sake of simplicity, the following system of equations is obtained using the averaged Eqs. (1)-(4)

$$\frac{\partial \rho S}{\partial t} + \frac{\partial \rho J}{\partial z} + Q = 0, \tag{13}$$

$$\frac{\partial J}{\partial t} + \frac{1}{\rho}\frac{\partial}{\partial z}\rho u J - \Pi = 0, \tag{14}$$

$$\frac{\partial \Pi}{\partial t} + \frac{1}{\rho}\frac{\partial}{\partial z}\rho u \Pi + N^2 J = 0, \tag{15}$$

where Q is the rate of entrainment of mass from the surrounding gas in the jet

$$Q = 2\pi R(z,\ t)\,\rho\left[v_r(z,\ R,\ t) - \frac{\partial}{\partial t}R(z,\ t)\right]; \tag{16}$$

$v_r(Z,\ R,\ t)$ is the radial velocity component at the jet boundary; also

$$J = uS, \tag{17}$$

$$\Pi = \Omega S. \tag{18}$$

The system in Eqs. (13)-(15) may be closed using the assumption – with a sound basis in the theory of turbulent jets – that the growth rate of the jet radius is determined by the pulsational velocity, which, in turn, is proportional to the mean jet velocity over the cross section [1]

$$\frac{\partial R}{\partial t} + u\frac{\partial R}{\partial z} = c|u|, \tag{19}$$

where c is an empirical constant, of the order of 0.1.

It is evident that Eq. (19) is inapplicable in the region of the upper and lower boundaries of the thermic. Therefore, the results obtained may be valid only for sections of the thermic sufficiently far from its boundaries.

The system in Eqs. (13)-(15) and (19) may evidently be regarded as a variant of the integral methods of turbulent-jet theory [1].

First consider the case of neutral atmospheric stratification (N = 0). For thermics with vertical dimensions much less than the height of the homogeneous atmosphere, the density and external temperature may be assumed constant, which allows Eqs. (14) and (15) to be written in the following form in this case

$$\frac{\partial J}{\partial t} + \frac{\partial uJ}{\partial z} = \Pi, \tag{20}$$

$$\frac{\partial \Pi}{\partial t} + \frac{\partial u\Pi}{\partial z} = 0. \tag{21}$$

If, at the moment of its formation, the thermic is practically homogeneous in the vertical direction, this homogeneity is retained for some time t_*. At times less than t_*, the spatial derivatives in Eqs. (20) and (21) may be neglected, and an accurate solution is obtained. At times much greater than the time to obtain establish buoyancy conditions $t_n \sim [J_0/\Pi_0]$:

$$R = R_0(1 + \tau^{-2}t^2)^{1/3}, \tag{22}$$

$$\Omega = \Omega_0 R_0^2 R^{-2}, \tag{23}$$

$$u = \Omega t, \tag{24}$$

where

$$\tau^2 = 2R_0(3c\Omega_0)^{-1}. \tag{25}$$

In the spatially inhomogeneous case, the solution in Eqs. (22)-(24) may be valid with a sufficiently weak dependence of $\Omega_0(z)$ and $R_0(z)$ on z. In this case, the flow characteristics will depend on z as a parameter. The time t_* may be obtained using Eqs. (22)-(24) to estimate the terms in Eqs. (19)-(21) with spatial derivatives

$$t_* \sim \tau\left(\frac{c\Delta z}{R_0}\right)^{3/2}, \tag{26}$$

where Δz is the characteristic scale of variation in the parameters of the initial distribution along the jet axis. It is readily evident that t_* is the time of displacement of a liquid particle by an amount ∿Δz.

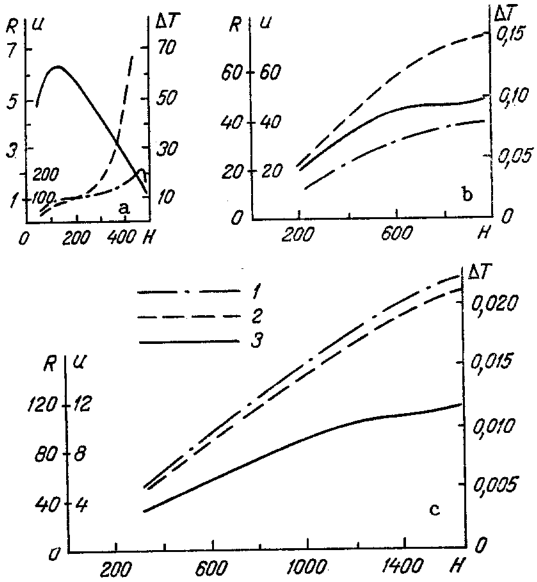

Fig. 1. Evolution of the distribution of the velocity 1, excess temperature 2, and radius 3 of the thermic along the vertical coordinate. Results of numerical solution of Eqs. (19)-(21) when T = 300 K, g = 10^4 m·sec^{-2}: a) initial distribution; b) t = 8 sec; c) 32 sec. H, R, m; u, m·sec^{-1}.

At times $t \gg t_*$, when the flow becomes significantly inhomogeneous the determining parameter of the problem is the integral (over the volume) buoyancy $\tilde{\Pi}$, which is conserved over time according to Eq. (5). The self-similar solution of Eqs. (19)-(21) is sought in the form in [5], defined by the relations

$$\frac{z}{\xi}=\frac{R}{r(\xi)}=\frac{2ut}{v(\xi)}=\frac{2\Omega t^2}{\omega(\xi)}=\tilde{\Pi}^{1/4}\rho^{-1/4}t^{1/2}. \tag{27}$$

For the new unknowns r, v, w, the following system of equations is obtained from Eqs. (19)-(21) on taking account of Eq. (27)

$$vr^2-\xi\frac{\partial}{\partial\xi}vr^2+\frac{\partial}{\partial\xi}v^2r^2=2\omega r^2, \tag{28}$$

$$\omega r^2+\xi\frac{\partial}{\partial\xi}\omega r^2-\frac{\partial}{\partial\xi}\omega r^2 v=0, \tag{29}$$

$$r-\xi\frac{\partial r}{\partial\xi}+v\frac{\partial r}{\partial\xi}=c\cdot|v|, \tag{30}$$

the solution of which takes the form

$$r=c\xi, \tag{31}$$

$$v=\omega=\begin{cases}\xi \text{ when } \xi\leqslant\xi_{\max},\\ 0 \text{ when } \xi>\xi_{\max}.\end{cases} \tag{32}$$

The value $\xi_{max} = 2(2\pi c^2)^{-1/4}$ is found from the condition of conservation of the total buoyancy $\tilde{\Pi}$.

An analogous problem regarding the dynamics of a thermic when N = 0 in a more general formulation, taking acount of the radial parameter distribution, was solved in the jet approximation in [8, 9].

The axial velocity and temperature of the thermic obtained in [8, 9] increases practically linearly with height, which agrees with Eq. (32). However, according to [8, 9], the thermic radius does not depend on the height, in contrast to Eq. (31); this is associated with the assumption that the viscosity is independent of the longitudinal coordinate [8, 9] and may correspond to a situation in which the turbulence of the thermic is determined by the turbulence of the atmosphere. In the present work, Eq. (19) actually assumes that the turbulence of the thermic is generated by its motion relative to the surrounding air, and the turbulent viscosity is proportional to the product of the radius and its velocity.

The results of numerical solution of Eqs. (19)-(21), reflecting the process by which the system approaches self-similar flow conditions in Eqs. (31) and (32), are shown in Fig. 1.

Proceeding to the investigation of stable stratification of the atmosphere ($N^2 > 0$), consider the case of a thermic that is practically homogeneous over the height. Discarding terms with spatial derivatives in Eqs. (14), (15), and (19), it is found that

$$J(t) = \tilde{u}S_0 \sin[N(t - t_0) + \varphi], \tag{33}$$

$$\Pi(t) = N\tilde{u}S_0 \cos[N(t - t_0) + \varphi], \tag{34}$$

$$R(t) = R_0 \left\{1 + \tau_1^{-1} \int_0^{t-t_0} |\sin(Nt' + \varphi)|\, dt'\right\}^{1/3}, \tag{35}$$

$$\operatorname{tg}\varphi = NJ_0\Pi_0^{-1}, \tag{36}$$

$$\tilde{u}^2 = u_0^2 + \left[\frac{g}{N}\left(\frac{\Delta T_0}{T} - \frac{\Delta\mu_0}{\mu}\right)\right]^2, \tag{37}$$

$$\tau_1 = R_0(3c\tilde{u})^{-1}. \tag{38}$$

The time of onset of evolution t_0 may be related to the velocity of motion of the heated-gas source V_p

$$t_0 = \int_{z_0}^{z} V_p^{-1}(z')\, dz'. \tag{39}$$

The corrections δJ, $\delta\Pi$, δR to the solution in Eqs. (33)-(35) produced by the inhomogeneity may be calculated from perturbation theory

$$\delta J = -\int_{t_0}^{t} N^{-1}\left(F_2 + \frac{\partial F_1}{\partial t'}\right) \sin[N(t - t')]\, dt', \tag{40}$$

$$\delta\Pi = \int_{t_0}^{t} N^{-1}\left(N^2F_1 + \frac{\partial F_2}{\partial t'}\right) \sin[N(t - t')]\, dt', \tag{41}$$

$$\delta R = R^{-2}(t) \int_{t_0}^{t} \left[-u\frac{\partial R}{\partial z} + cR^{-2}\delta J\right] R^2 dt', \tag{42}$$

where

$$F_1 = \rho^{-1}\frac{\partial}{\partial z}\rho u J; \tag{43}$$

$$F_2 = \rho^{-1}\frac{\partial}{\partial z}\rho u \Pi. \tag{44}$$

Estimating the integrals in Eqs. (40)-(42) for times much greater than the period N^{-1}, the following expression is obtained for the amplitude ratio

$$\frac{|\delta J|}{|J|} \lesssim \frac{|\delta\Pi|}{|\Pi|} \sim \frac{\tilde{u}N^{-1}}{\Delta z} + \frac{R(t)}{3cz(N)}, \tag{45}$$

where Δz is the resulting scale of variation in the parameters along the jet due to inhomogeneity of the atmosphere, the height dependence of the source characteristics, and the time of onset of evolution

$$\Delta z^{-1} = z^{-1}(\rho) + z^{-1}(N) + z^{-1}(\tilde{u}) + z^{-1}(\varphi) + NV_p^{-1} + (V_p\tau_1)^{-1}, \tag{46}$$

z(f) is the characteristic scale of variation in f; and

$$\frac{|\delta R|}{R} \sim \tilde{u} N^{-1} \left(\frac{\tau_1}{t}\right)^{\frac{2}{3}} [z^{-1}(R_0) + z^{-1}(\tilde{u}) + z^{-1}(N)] + \frac{|\delta J|}{Nt\,|J|}. \quad (47)$$

From the requirement that the ratio in Eq. (45) be small in comparison with unity, it follows that the conditions of applicability of Eqs. (33)-(35) are that the displacement of the liquid particle in the oscillation period be small in comparison with the scale of inhomogeneity Δz, and that the jet radius be small in comparison with cz(N). It may be deduced from the last condition that the validity of Eqs. (33)-(35) is restricted to times less than

$$t_* = \tau_1 [3cz(N) R_0^{-1}]^3. \quad (48)$$

When R_0 = 100 m, $\tilde{u}$ = 100 msec^{-1}, c = 0.1, and z(N) = 10^4 m, Eqs. (38) and (48) give t_* of the order of 10^5 sec. Apart from those obtained from Eq. (45), no further constraints follow from Eq. (47).

The singular role of the scale z(N) in estimating the ratio in Eq. (45) arises in that, over time, the oscillations of adjacent liquid particles with different frequencies are transformed from cophase to counterphase, which leads to the appearance of spatial gradients that grow indefinitely (within the framework of perturbation theory) over time. The inhomogeneity of the other parameters makes a limited contribution to the value of the spatial derivatives.

NOTATION

t, time; x_i, i-th coordinate; v_i, i-th projection of the velocity; z, u, vertical coordinate and velocity; v_r, radial component of velocity; V, volume; t_*, time of existence of quasi-homogeneous conditions, defined in Eqs. (26) and (48); τ and τ_1, characteristic times, defined in Eqs. (25) and (38); $\tilde{u}$, quantity defined in Eq. (37); Δz, resulting scale of parameter variation along the thermic axis; z(f), characteristic scale of variation in f; ρ, density; T, temperature; κ_m, gravimetric concentration of m-th impurity; μ, molecular weight; $\Delta\rho = \rho - \rho_\infty$; $\Delta T = T - T_\infty$; $\Delta\kappa_m = \kappa_m - \kappa_{m\infty}$; $\Delta\mu = \mu - \mu_\infty$; c_p, specific heat at constant pressure; g, acceleration due to gravity; N, buoyancy frequency, defined in Eq. (10); R and $S = \pi R^2$, radius and cross-sectional area of thermic; $\tilde{J}$ and $\tilde{\Pi}$, integral (over the thermic volume) vertical momentum and buoyancy, defined in Eqs. (7) and (8); Ω, specific buoyancy, defined in Eq. (9); J = uS; $\Pi = \Omega S$; δR, δJ, $\delta\Pi$, corrections to R, J, Π due to deviations from homogeneity; ξ, r, v, ω, dimensionless analogs of z, R, u, Ω; Q, rate of entrainment of the mass of surrounding air into the jet; c, constant of turbulent mixing; V_p, velocity of propagation of heated-gas source; F_1, F_2, functions defined in Eqs. (43) and (44); t_0, initial time; <...>, mean over the turbulent pulsations; a bar above a symbol denotes the mean over the thermic cross section. Indices: 0, value at time t_0; ∞, parameters of atmosphere at the same height as the given point of thermic.

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CALCULATION OF THE FLOW OF A POLYDISPERSED SYSTEM OF PARTICLES

A. K. Azhibekov UDC 532.529

We discuss a model which can be used to compute the velocities and temperatures of solid and liquid particles in the presence of collisions and coagulation.

Calculation of the flow of a polydispersed system of solid and liquid particles has been considered by numerous researchers in recent years ([1-4] and others). This is because of the great abundance of multiphase systems in nature (aerosol processes in the atmosphere) and in technology (flow of a gas with suspended particles in a nozzle, carburetion processes in combustion chambers, etc.).

We consider the steady one-dimensional flow of a polydispersed system of solid and liquid particles in a gas. As an example, we consider the motion of a three-phase system consisting of a gas, solid dust particles, and water droplets, where the latter two phases are suspended in the gas. The system flows in a channel of variable cross section (a venturi serving as a dust trap). The fundamental problem is to determine the parameters of the three-phase mixture and to calculate the degree of precipitation of solid particles into the liquid droplets, the pressure drop, and the temperature decrease of the carrier medium. Obviously in order to be able to solve this problem, we must know the size distribution functions of the solid particles $dN_1 = f(\delta_1)d\delta_1$, m^{-3} and the liquid droplets $dN_2 = f(\delta_2)d(\delta_2)$, m^{-3} under a variety of conditions. The most significant factor for these distributions is the collision and coagulation of particles of different fractions. There are three types of collisions for the problem considered here: a) solid particle—solid particle collisions; b) liquid droplet—liquid droplet collisions; c) solid particle—liquid droplet collisions.

From the estimates of [4] we assume that collisions between solid particles do not lead to their coagulation; these collisions are then termed ineffective. On the other hand, the other two types of collisions are effective, and each collision leads to complete coagulation of the particles. We note that collisions of the last two types are the basic process of dust capture and therefore the degree of purification of the exhaust gas depends on the frequency and effectiveness of these collisions. Strictly speaking (as shown in [1]) a not uncommon case is when the collision leads to fragmentation of the particles, as well as partial coagulation. Processes of this type are not considered at all in the present paper.

A collision leads to an excess (or deficit) of momentum and energy of the newly formed (as a result of coagulation) particle. Therefore the velocity and temperature of the newly formed particle can differ significantly from the velocity and temperature of particles of the same size but not subjected to perturbing factors. In addition, it is very important in the solution of problems of this kind to take into account the fact that solid particles, which earlier had precipitated into liquid droplets, can return to the flow after the droplets have completely evaporated.

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